Arrow’s Impossibility Theorem and Distributive Justice Theories

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Abstract

This paper explores the relationship between Kenneth Arrow’s impossibility theorem and distributive justice theories. Arrow’s theorem is an important result in social choice theory. It tells us that social choice is impossible since the conditions necessary for it cannot all be simultaneously satisfied. Distributive justice theories, on the other hand, aim to provide sound principles for the distribution of burdens and benefits in a given society. These principles serve as norms that political and social institutions must use to justly allocate goods, welfare, and other resources to their constituents. This paper aims to argue that since a social choice is impossible given Arrow’s theorem, it follows that it is impossible to choose the principle for a just distribution that a given society must prefer since such a choice is a social choice.

Keywords: Arrow’s impossibility theorem; distributive justice theory; social choice theory

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ISSN: 2467-5784
Introduction

The Philippines has been one of the pioneers in developing land reform programs in the world. This is evidenced by many significant land reform laws that the country has enacted, from the 1963 Agricultural Land Reform Code to the 2009 Comprehensive Agrarian Reform Program Extension with Reforms (CARPer).¹ The basic motivation of these laws is to institute a more just land redistribution scheme that would benefit the underprivileged farmers who toil the affluent hacienderos' piece of land in place of the hacienderos themselves. Pursuant to the 1963 law, for example, the Philippine government abolished the tenancy system, which was deemed as abusive and unjust, and ignorant of the farmers’ welfare. In its stead, a leasehold system was established, where farmers paid fixed rentals to use farmland and earn from their labor rather than earning from a percentage of harvest as was practiced in the tenancy system.²

Academics, however, have been of two minds regarding such land reform programs. While some have been optimistic about them, others have been more pessimistic. For example, in Toward an Alternative Land Reform Paradigm: A Philippine Perspective, Yūjirō Hayami, Agnes R. Quisumbing, and Lourdes S. Adriano, complain that “the traditional Asian model of land reform (a non-communist model) is inappropriate for the Philippines, given the country’s unique agrarian structure.”³ Furthermore, James Putzel, in his A Captive Land: The Politics of Agrarian Reform in the Philippines, observes that “despite the central place given to agrarian reform on the political agenda in the Philippines throughout most of the 20th century, no significant land redistribution has occurred.”⁴

On the other hand, in his “State–Society Relations in Land Reform Implementation in the Philippines,” Saturnino Borras argues

¹ For a history of these programs, see the Department of Agrarian Reform’s write-up available online at <http://www.dar.gov.ph/about-us/agrarian-reform-history/>.
that “[c]ontrary to earlier pessimistic predictions, the Comprehensive Agrarian Reform Programme (CARP) in the Philippines has achieved significant success in land redistribution, although not quite matching the original claims of the state.”

Moreover, in his “Determinants and Consequences of Land Reform Implementation in the Philippines,” Keijiro Otsuka claims that “[u]nlike the experience of other developing countries, land reform in the Philippines has been successful in converting many of share tenants to land reform beneficiaries.”

The differing opinions surrounding land reform programs reflect the philosophical issue of whether such programs are moral and just; i.e. of whether the stakeholders of such programs (be it the landowners or farmers) are fairly treated and compensated. Questions about morality and justness in such a setting belong to a wider set of socio-political questions discussed in distributive justice theory, which is a branch of socio-political philosophy.

Distributive justice theories aim to provide moral principles that guide political and social institutions to justly allocate benefits and burdens in society. These principles serve as norms that govern how such institutions must allocate goods, welfare, and other resources to their constituents. Accordingly, the choice of a distributive justice principle must benefit society as a whole. As social choices, however, distributive justice theories must come to terms with an important result in social choice theory, a branch of welfare economics aimed at providing mathematical models of collective decisions. This result is Arrow’s impossibility theorem.

The impossibility theorem was proposed by the American mathematician and economist, Kenneth Arrow. Along with John R. Hicks, he was awarded the Nobel Prize for Economics “for their pioneering contributions to general economic equilibrium theory and welfare theory.” Arrow’s work was “one of the first impossibility

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theorems outside of pure mathematics” and “became a starting point of social choice theory”.10

In a nutshell, Arrow’s theorem tells us that social choice is impossible since the conditions necessary for it cannot be consistently satisfied. Some interpreters of the theorem identify four such conditions. These are universality, independence (of irrelevant alternatives), Pareto efficiency, named after the Italian economist, Vilfrido Pareto, and non-dictatorship.11

This paper shows that Arrow’s impossibility theorem has a negative impact on distributive justice theories. Since social choice is impossible, then so is distributive justice. A more nuanced formulation of the argument and a case for its soundness will be developed in the next three sections. The final section summarizes the line of reasoning advanced in the previous sections, and addresses possible objections that could be raised against the main argument.

The Argument from Arrow’s Impossibility Theorem

The main argument of this paper might be labelled as the argument from Arrow’s impossibility theorem. It could be cashed out in this simple syllogism:

1. Choosing a distributive justice principle is a social choice.
2. Social choice is impossible.

Therefore,

3. Choosing a distributive justice principle is impossible.

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This argument is deductively valid. It is an instance of the valid syllogistic form, BARBARA; to wit: All S is M. All M is P. Thus, all S is P. An argument’s validity, however, does not guarantee the truth of its conclusion. To guarantee its truth, what is also required is that the supporting premises are true or at least are plausible, which amounts to showing its soundness.

Let us first examine premise 1. Premise 1 implies that deciding on what distributive justice principle applies in allocating benefits in society, for example, belongs to a wider category of collective choices. These are choices generated from personal preferences of members of a group.

Not all group choices, however, are about distributive justice principles; some might not even raise any distribution issue at all. For instance, when a group deliberates on where and when to eat dinner, the group’s eventual choice is about their dining preferences. Such a choice does not necessarily involve concerns about distributive justice. If they deliberate on how they divide the bill after dinner, however, then their deliberation and eventual choice does involve such an issue. This is so since they would have to decide the right way of distributing responsibilities.

Let us make the case for premise 1 more explicit. When a group of individuals deliberate on how to best distribute a particular good, their deliberation implies that each member has a sense of the right way of distributing benefits and burdens within the group. In effect, each one already has a distributive justice principle in mind. Since their eventual choice is brought about by the individual choices, their eventual group choice will be a kind of social choice. Thus, choosing a principle for a just distribution of burdens and benefits within a given society is a kind of social choice.

Let us now consider premise 2: that any social choice is impossible. This key premise is Arrow’s impossibility theorem, and the argument presented above has this theorem as its major premise. Thus, if this theorem holds, the argument’s conclusion follows. Here is a preliminary sketch of Arrow’s proof.

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13 By definition, an argument is sound if it is valid and has true premises. Cf. ibid., 31.
14 The fourth section of this paper will present a more detailed, albeit simple version of Arrow’s proof.
Suppose that there is a group of individuals whose members have a set of preferences. Suppose further that each member may rank their preferences in some rational ordering. To arrive at a group choice, four conditions must be present. These conditions are universality, independence, Pareto efficiency, and non-dictatorship.\(^\text{15}\) Universality implies that each individual’s ranked preferences may be aggregated into a single group preference. Independence entails that, except for the initial set of alternatives chosen from, no other alternative must be considered in the individual ranking. Pareto efficiency means that whatever the aggregated individual preference ranking might be, it is strictly the group’s preference. Finally, non-dictatorship entails that no single individual preference controls the group’s preference. What Arrow has shown is that these conditions cannot all be simultaneously satisfied since there will always be exactly one individual, a dictator, whose preference dominates others. Thus, social choice is impossible. Alternatively, a social choice would be possible if the choice made is “either imposed or dictatorial.”\(^\text{16}\)

We have now a *prima facie* case for the premises of the main argument above, which the succeeding two sections will develop. Let us begin with premise 1: that choosing a principle for a just distribution of burdens and benefits within a given society is a social choice.

**Distributive Justice Principles as a Social Choice**

Distributive justice issues emerge when members of a given society put forward conflicting claims of their share of the benefits in that society. Since these claims conflict, each individual’s claim is often left unsatisfied.\(^\text{17}\) There are two aspects of distributive justice that we could highlight here. First, distributive justice is about the allocation of benefits and burdens within a given society. Second, there are stakeholders that would either benefit or be burdened by such allocation. The goal of any distributive justice theory is to provide moral and just principles that guide the decision-making process for such allocations.

Let us again consider the case of land reform programs as an illustration. There are three central stakeholder groups in this type of

\(^{15}\) See fn. 11.

\(^{16}\) Cf. Arrow, "A Difficulty in the Concept of Social Welfare," 342.

case: the *hacienderos* who own the land, the farmers who use the land, and the government that arbitrates and makes policies. Each group has a claim on how to best go about land reform. Perhaps, some *hacienderos* might be against the whole idea of land reform since they will be most burdened by it. Most farmers, of course, will be for it since they will greatly benefit from it. The claims of *hacienderos* and farmers are surely in conflict, and it is left to the government to secure and promote the welfare of both parties. Even the government itself, however, might have a wider stake in the issue. It needs to think about how such a proposed land reform policy might affect the general economic welfare of the entire country. These considerations, then, point to the issue of the best distributive justice principle that must be employed in this case. Arguably, this case generalizes to other types of distribution cases as well.

The distributive justice theory literature offers several principles to achieve a moral and just allocation of benefits and burdens within society. According to Julian Lamont and Christi Favor’s *Stanford Encyclopedia of Philosophy* entry on the subject, six are considered mainstream. They are the strict egalitarian approach, the difference principle, resource-based approach, welfare-based approach, desert-based approach, and the libertarian approach.¹⁸ Let us consider the basic ideas of each of these approaches in turn.¹⁹

The strict egalitarian approach tells us that to achieve a just distribution, “every person should have the same level of material goods (including burdens) and services.”²⁰ This approach implies a kind of radical equality, such that a just distribution implies that each and every person should have the same amount of benefits and burdens. Thus, in the case of land reform, both the *hacienderos* and the farmers should have an equal share of the land and of whatever will be produced in it.

There are several criticisms that could be raised against the strict egalitarian approach. Here we focus on two such criticisms: the index problem and the time-frame problem.²¹ The index problem raises

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¹⁹ The forthcoming discussion is not meant to be exhaustive and in-depth. The motivation here is simply to have a conceptual handle of the basic ideas of the different approaches to distributive justice. For a more detailed discussion of the literature, see Allingham, *op. cit.*

²⁰ Lamont and Favor, *op. cit.*, sec. 2.

²¹ Ibid.
a question about the adequate way of measuring sameness of the level of benefits and burdens. For example, in a land reform program, how should we divide the shares of one haciendero and nine farmers? If we follow the strict egalitarian approach, then their shares should be divided equally. But what does “equality of shares” imply? Suppose that 10,000 cavans of rice were produced by the land, does this mean that the haciendero will have 5,000, while the farmers will divide the rest amongst themselves? Or does it mean that each of them will receive 1,000 cavans? Furthermore, who decides which interpretation must hold?

The time-frame problem, on the other hand, raises the question about when the distribution should take place. For example, suppose that the haciendero and the nine farmers agree to each receive 1,000 cavans of the 10,000 cavans produced this harvest season. Would such an arrangement hold for the next harvest season or the harvest season after that? How about the harvest season fifty years after, when the original parties in agreement have long been gone? Time is an integral factor in many of our decisions, and this is highlighted by this problem.

Let us next consider the difference principle proposed by the American political philosopher John Rawls. Rawls presents two principles of justice; namely,

Each person has an equal claim to a fully adequate scheme of equal basic rights and liberties, which scheme is compatible with the same scheme for all; and in this scheme the equal political liberties, and only those liberties, are to be guaranteed their fair value.

Social and economic inequalities are to satisfy two conditions: (a) They are to be attached to positions and offices open to all under conditions of fair equality of opportunity; and (b), they are to be to the greatest benefit of the least advantaged members of society. 

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22 Ibid.
23 Ibid.
These two principles amount to one basic principle for a just distribution of benefits and burdens within a given society. Any distribution scheme would do so long as it benefits the least advantaged group in a given society. Thus, in the case of the *haciendero* and the nine farmers, Rawls’ view implies that the nine farmers ought to have a greater share of the produce than the *haciendero* since they are the disadvantaged group between the two.

This result, however, is not without problems and, as Lamont and Favor points out, “[b]ecause there has been such extensive discussion of the Difference Principle in the last 40 years, there have been numerous criticisms of it.” Here we highlight only one such criticism, namely, that Rawls’ principle generates material inequalities, which would result in further injustices in the long run.

Consider again the case of the *haciendero* and the nine farmers. Given the Rawlsian distribution scheme, the *haciendero* should only receive a small share of the harvest, perhaps, a tenth of a tenth, while the rest will be shared amongst the nine farmers. This means that the *haciendero* will only receive 100 of the 10,000 cavans, while 9,900 will be divided amongst the farmers. This disparity might first seem reasonable given that the farmers are clearly the disadvantaged group. If we were to employ this scheme over time, however, matters would be much different.

Suppose that we hold constant both the Rawlsian distribution scheme and the number of cavans produced per harvest season. Then, *ceteris paribus*, over a period of ten harvest seasons, the *haciendero* would only receive 1,000 cavans as opposed to the nine farmers’ 99,000. Surely, this material disparity is a kind of injustice. The *haciendero*, the primary investor in this case, will be losing his or her initial investment. As such, he or she would eventually be incapable of sustaining the business over time.

A Rawlsian reply to this criticism might take a line from Lawrence Croker’s “Equality, Solidarity, and Rawls’ Maximin.” There is an apparent injustice in the *haciendero*’s case only if all that matters to us is material equality. If we also put a premium on other values, like solidarity, which “cause people to seek out situations in which there are material inequalities”...
strong feelings of cooperation, mutual identification, and similarity of status and position," then the value of material equality diminishes.\textsuperscript{30}

Let us now consider two approaches proposed by two other influential American political philosophers; namely, Richard Dworkin's resource-based approach, which is a version of luck egalitarianism, and Robert Nozick's entitlement theory, which is a version of the libertarian approach.\textsuperscript{31} Let us examine each in turn.

Dworkin's approach assumes that people initially have an equal share of the resources. Due to their own choices, however, they end up having unequal economic successes.\textsuperscript{32} For him, a just distribution of resources only happens once. While the turnout of such a distribution may cause inequalities later on, the initial distribution is not to be blamed since each individual is solely responsible for how he or she uses his or her share of resources.\textsuperscript{33}

On the other hand, according to Nozick,

If the world were wholly just, the following inductive definition would exhaustively cover the subject of justice in holdings:

(a) A person who acquires a holding in accordance with the principle of justice in acquisition is entitled to that holding.

(b) A person who acquires a holding in accordance with the principle of justice in transfer, from someone else entitled to the holding, is entitled to the holding.

(c) No one is entitled to a holding except by (repeated) applications of (a) and (b).

The complete principle of distributive justice would imply that everyone is entitled to the holdings they possess under the said distribution.\textsuperscript{34}

Nozick's entitlement theory is founded on the principles of a free-market system. Despite having no real fixed normative principles of how-to's, it has principles that explain the market's behavior through the interaction of market forces. These explanatory principles are not norms that need to be strictly followed. They serve as a way to explain how the market behaves. Accordingly, Nozick's view implies that any

\textsuperscript{30} Ibid., 263.
\textsuperscript{31} Cf. Lamont and Favor, \textit{op. cit.}, secs. 4 and 7.
\textsuperscript{33} Lamont and Favor, \textit{op. cit.}, sec. 4.
\textsuperscript{34} Ibid., sec 7 quotes this from Robert Nozick, \textit{Anarchy, State, and Utopia}, (New York: Basic Books, 1974), 151.
sort of distribution is just so long as we abide by the three principles above. That is, someone is entitled to have some benefits or burdens only because they are his or hers in the first place.  

How would these two approaches account for the case of the *haciendero* and the nine farmers? Unfortunately, Dworkin’s view has almost nothing to say about it. Suppose that the *haciendero* and the nine farmers agree upon a 60-40 distribution scheme. If we go by Dworkin’s approach, then we must assume that there is an initial equal distribution of resources, like land, people’s abilities, and so on, and people’s successes and failures are determined solely by their own free choices. But even if we grant these two things, we still have no reason why the preferred 60-40 scheme ought to be adopted since it would just be a matter of luck why the initial distribution happened the way it did and why people would choose the course of action the way they would. This indicates one of the weaknesses of the resource-based approach; namely, it lacks “practical ways the economic systems can be refined to track responsibility while mitigating certain types of pure luck.”

Depending on the sort of distribution proposed by the *haciendero*, Nozick’s approach, on the other hand, would have a predicament similar to Dworkin’s or to Rawls’. If Nozick is right, then since the *haciendero* is the rightful owner of the land, it follows that despite any effort that farmers put into production, the *haciendero* will be entitled to whatever the land produces. As such, the *haciendero* would have the exclusive right to determine what the farmers’ share should be. This, however, might lead to an injustice towards the farmers if the distribution is unfavorable to them. Thus, like Rawls’ approach, Nozick needs to account for injustices that his view might imply. On the other hand, if the distribution is favorable to all parties concerned, then Nozick’s approach, like Dworkin’s, needs to provide the grounds for such a distribution.

Finally, let us turn to two other standard approaches to distributive justice; namely, the welfare-based approach and the desert-based approach. The welfare-based approach is grounded on the concept of welfare. Though the concept is ambiguous and its interpretation contentious, we could still have a rough idea of the principle it offers. It implies that a just distribution of benefits and

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35 Lamont and Favor, *op. cit.*, sec. 7.
36 Ibid.
37 Ibid., secs. 5 and 6.
burdens should maximize both personal and public welfare of society and its members.\textsuperscript{38}

Suppose that, \textit{a la} classic utilitarianism, we define welfare in terms of happiness. The welfare-based approach entails that we should allocate goods according to how they maximize each person’s happiness, which, in turn, collectively contributes to the overall happiness of society.\textsuperscript{39} Thus, if a 70-30 sharing promotes the happiness of all parties concerned, then that sort of distribution scheme ought to be adopted. Otherwise, some other scheme must be used. Given its machinery, the welfare-based approach provides an easy solution to most distribution issues.

The desert-based approach, on the other hand, tells us that a just distribution must be according to what each person \textit{deserves}.\textsuperscript{40} Like the concept of welfare, the concept of desert is also ambiguous. Different versions of the desert-based approach depend on how this concept is defined. For example, some philosophers, like David Miller, think that a person deserves some benefits if his or her action produces something valuable to society. Others, like Heather Milne, suggest that one deserves such benefits because of the effort that he or she has exerted to achieve a particular end.\textsuperscript{41} Still others, like James Dick, contend that the benefits that one deserves correspond to the monetary gains or losses incurred by that individual in a social transaction.\textsuperscript{42}

Suppose that we characterize what it is to deserve something in terms of the effort one puts into an activity. The corresponding desert-based approach, then, implies that, in the case of a distribution of goods, the person who exerted the best effort must deserve a larger share than those who exerted less. Looking back to the case of the \textit{haciendero} and the farmers, since the latter exerted greater effort than the former, they must deserve a larger share of the harvest than the \textit{haciendero}. As such, like the welfare-based approach, the desert-based approach has enough conceptual resources to provide a just principle of distribution.

Despite their deliverables, however, both the welfare-based and the desert-based approaches will still face some of the problems that

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\textsuperscript{38} Ibid., sec. 5.
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the other approaches faced. In particular, both would have to face the index problem that the strict-egalitarian approach faced. Furthermore, both would have to also face the applicability problem that Rawls’, Dworkin’s and Nozick’s approaches also faced. Let us discuss these two problems in turn.

If we use either the concept of welfare or desert as the basis of our distribution scheme, then we should have an adequate measure of either concept. For example, if the farmers must have a greater share of the harvest than the *haciendero* because they either exerted more effort in producing it or they will be happier if such share was given, then, in principle, there is a way of measuring the amount of effort exerted or the amount of happiness that will be attained. But, as Lamont and Favor argue, “there is no conceptually adequate way of calibrating such a measure among individuals.”\(^{43}\) This is so since what it means for someone to exert more effort or be happier than someone else depends on some other related property.

For example, if the farmers’ and *haciendero’s* efforts are measured in terms of the amount of *physical* work they have exerted, then it is true that the farmers exerted more effort than the *haciendero*. If, on the other hand, the measure involves *mental* effort exerted to plan the project, then, perhaps, the judgment of who exerted more effort will be the other way around. The same line of reasoning here applies to the concept of welfare. Thus, since neither concept admits an adequate measure, they cannot be the basis for our distribution scheme.

Relatedly, both the welfare-based and the desert-based approaches might yield injustices in the long run. For example, either approach might favor the farmers over the *haciendero* in distributing the cavans this harvest season. But, as in the case against Rawls’ approach, if we employ this sort of distribution scheme over time, the *haciendero* might lose all his or her investments and, thus, would be disenfranchised.

The six distributive justice theories discussed above provide a picture of what a theory of distributive justice aims to achieve. As we have seen, each approach aims to provide a moral and just principle. for a distribution scheme. This implies that a theory of distributive justice offers a principle of just distribution that serves as the foundation of any decision-procedure designed to allocate benefits and burdens to individual members of a given society.

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\(^{43}\) Lamont and Favor, *op. cit.*, sec. 5.
It is true that each individual member may have a different preference (or a concept of justice) for the allocation of these benefits and burdens. As such, it is possible that no single decision-procedure that can be agreed upon by all the stakeholders. It follows that any decision-procedure that defines a principle of just allocation of benefits and burdens should come to terms with the gap between individual preferences and a collective choice. This is a gap that social choice theory aims to account for.

David Mayston offers a more precise description of what social theory is all about. He writes,

The idea of social choice, and the theory which surrounds it, is basically that of the generation of social choices from the preferences of different individuals in the society, in such a way that the social choices possess a number of well-specified properties.  

This characterization implies that social choice theory is concerned with issues where the relationship between individual personal preferences and social choice is a factor. Admittedly, this characterization is broad. Too broad, in fact, that it may include issues from different ends of the intellectual spectrum: from issues that are highly academic, like issues dealt with in political theory, sociology, and economics to mundane everyday issues, like how to slice a cake, how to assign daily chores, or how to plan a party. The broadness of social choice theory, however, is not a weakness, it is a strength.

Sen agrees with this.  Given that there are different ways that individual preferences and collective choices interact, there must be different types of collective choices. He identified three such types:

(i) Committee decision: A committee has to choose among alternative proposals for action on the relative merits of which the members hold different views.
(ii) Social welfare judgement: A person want (sic) to make a judgement whether a certain change will be better for the society, some members of which will gain from the change while others will lose.

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With the foregoing discussion, we can now present the argument why choosing a principle for a just distribution of burdens and benefits within a given society is a social choice. Issues of distributive justice implies that stakeholders deliberate on what a right, morally just distribution scheme must hold in certain cases. This kind of deliberation implies an interaction between each stakeholder's personal preference of a right distribution scheme and the eventual choice that all the stakeholders involved would agree upon. Since social choice is any kind of interaction between individual preferences and collective choices, it follows that choosing a principle for a just distribution of burdens and benefits within a given society is a kind of social choice.

**Arrow's Impossibility Theorem**

In his 1950 paper, “A Difficulty in the Concept of Social Welfare,” Arrow puts forward the following theorem:

If we exclude the possibility of interpersonal comparisons of utility, then the only methods of passing from individual tastes to social preferences which will be satisfactory and which will be defined for a wide range of sets of individual orderings are either imposed or dictatorial.\footnote{Arrow, “A Difficulty in the Concept of Social Welfare,” 342.}

There are other ways of formulating Arrow's theorem. There is Joseph Lützen's formulation: “[i]t is impossible to design a welfare function (or a voting method) that satisfies some rather innocent looking requirements.”\footnote{Lützen, op. cit., 56.} There is also Mark Fey's: “[i]f a social preference function satisfies Unanimity and IIA (Independence of
Irrelevant Alternatives), then some individual is a dictator."\footnote{Mark Fey, “A Straightforward Proof of Arrow’s Theorem,” in \textit{Economics Bulletin} 34:3, (2014), 1793.} There is also Valentino Dardoni’s: [t]here is no SWF (social welfare function) which satisfies U (Unanimity), IIA (Independence of Irrelevant Alternatives), and ND (Non-dictatorship).\footnote{Valentino Dardanoni, “A Pedagogical Proof of Arrow’s Impossibility Theorem,” in \textit{Social Choice and Welfare} 18:1, (2001), 108.} Perhaps, it is Sen’s formulation that best captures not only the main crux of the theorem, but also its implications. He writes,

> [E]ven some very mild conditions of reasonableness could not be simultaneously satisfied by any social choice procedure, within a very wide family. Only a dictatorship would avoid inconsistencies, but that of course would involve: (1) in politics, an extreme sacrifice of participatory decisions, and (2) in welfare economics, a gross inability to be sensitive to the heterogeneous interests of a diverse population.\footnote{Sen, “The Possibility of Social Choice,” 351.}

Certain conditions are necessary to have a social choice from individual personal preferences. These conditions are (UD) universality or having an unrestricted domain, (PE) Pareto efficiency or unanimity, (IIA) independence of irrelevant alternatives, and (ND) non-dictatorship.\footnote{See fn. 11.} Arrow’s theorem implies that these conditions cannot simultaneously be satisfied without entailing a contradiction. If the dictatorship condition is dropped, however, the other three conditions can be satisfied. Thus, given UD, PE, and IIA, social choice is possible only if ND does not hold. Contrapositively, if ND does hold, then social choice is impossible. Before discussing further Arrow’s theorem, let us first appreciate the strategy that Arrow used to prove his theorem.

Arrow’s proof is a proof by contradiction (otherwise known as a \textit{reductio ad absurdum} proof). A proof by contradiction starts with an assumption that the proposition that aims to be proven is false. It ends when the assumption logically entails a contradiction since the original proposition is already proven true.\footnote{Cf. Harry Gensler, \textit{Introduction to logic}, 2nd edition, (London: Routledge, 2010), 153-157.} Arrow’s proof follows the same proof structure. It begins with a simple conditional proposition: if some proposition $F$ is true, then some other proposition $G$ is also true. Assuming that that conditional is false implies that $F$ is true and $G$ is
false. If this assumption leads to a logical contradiction of the form, $H$ and not-$H$, then it follows that if $F$ is true, then $G$ is true because to suppose otherwise leads to an absurd contradiction. Having this proof strategy in mind, let us now go through the steps of Arrow’s proof.\footnote{As much as possible, our presentation of Arrow’s proof will be less formal. For a more formal, yet simpler presentation of the proof, see Mark Fey, \textit{op. cit.}, and Dardanoni, \textit{op. cit.} For Arrow’s original proof, see his “A Difficulty in the Concept of Social Welfare,” 339-342 and his \textit{Social Choice and Individual Values}, ch. V.}

Arrow begins by stating the necessary conditions for social choice. Let us first consider universality.

UD: The social welfare function is defined for every admissible pair of individual orderings, $R_1, R_2$.\footnote{Arrow, “A Difficulty in the Concept of Social Welfare,” 336.}

Since social choices are generated from individual preferences, UD requires that any social welfare function, i.e., a function that maps individual preferences to possible social choices, must admit all possible ordered pairings of individual preferences. Nothing must restrict the number of possible ordered pairings of individual preferences that will be included in the pool of preferences from which a possible social choice may be drawn.

Let us further elaborate on what it means for an individual preference to be an ordered pairing. First, note that the preference relation is a two-placed relation. If we have three alternatives $x, y, z$, the preference relation can only range over exactly two of these alternatives. Thus, an individual may prefer $x$ over $y$, $y$ over $z$, or some other permutation of the two-placed relation.

Second, individual preferences are ordered pairings. They are a preference ranking relation that is reflexive (i.e., if someone prefers $x$ over $y$, then he or she prefers $x$ over $y$), asymmetric (i.e., if someone prefers $x$ over $y$, then he or she does not prefer $y$ over $x$), and transitive (i.e., if someone prefers $x$ over $y$, and prefers $y$ over $z$, then he or she prefers $x$ over $z$).\footnote{In social choice theory, this kind of relation is a weak preference relation. Cf. Dardanoni, \textit{op. cit.}, 108.} Ordered pairings are so defined in order “to produce ‘rational’ social preferences,” and to prohibit individual preferences that clearly do not make any sense, like when someone prefers $x$ over $y$ but also prefers $y$ over $x$, to be considered as social preferences.\footnote{Cf. \textit{List, op. cit.}, sec. 3.1.}

Given this, UD, then, implies the following. Suppose our society initially has two members, $a, b$, each of whom has alternatives, $x, y, z$, to...
choose from. Suppose that \( a \) prefers \( x \) over \( y \) and prefers \( y \) over \( z \), while \( b \) prefers \( y \) over \( z \) but prefers \( z \) over \( x \). UD implies that each of these individual preferences can be ranked such that a profile can be made for each set of individual preferences. Given their preference rankings, \( a \)'s ranking profile, \( R_1 \) places \( x \) as first, \( y \) as second, and \( z \) as third. On the other hand, \( b \)'s profile, \( R_2 \) places \( y \) as first, \( z \) as second, and \( x \) as third. Given the social welfare function, these profiles generate a possible choice of society as a whole. Suppose now that some other individual, \( c \) joins the society. Suppose further that \( c \)'s ranking profile, \( R_3 \) places \( x \) as first, \( z \) as second, and \( y \) as third. Given UD, this too must be included in the pool of possible social choices alongside \( R_1 \) and \( R_2 \).

Let us now turn to the Pareto efficiency condition and see how it relates with UD.

**PE**: If an alternative social state \( x \) rises or does not fall in the ordering of each individual without any other change in those orderings and if \( x \) was preferred to another alternative \( y \) before the change in individual orderings, then \( x \) is still preferred to \( y \).\(^{58}\)

PE implies that if we aggregate all individual ranking profiles and such aggregation yields an alternative that is generally preferred over the other alternatives, then the social welfare function must strictly place that preferred alternative over the others. This preferred alternative becomes society’s choice.\(^{59}\) This follows since, by definition, the social welfare function generates social choices from individual preferences.

Consider again the individual ranking profiles \( R_1, R_2, R_3 \). Aggregating them places \( x \) over the other alternatives. Since the aggregate yields such a preference, it follows, given PE, that society as a whole prefers \( x \) over \( y \).

Now suppose that some other individual, \( d \) joins our society, and his or her ranking profile, \( R_4 \) places \( z \) as first, \( x \) as second, and \( y \) as third. Given UD, we must include \( R_4 \) in our pool of a possible social choice. But even so, the social welfare function will still show that society as a whole prefers \( x \) over \( y \). This result follows since, once aggregated, all individual ranking profiles, including \( R_4 \), still place \( x \) over the other alternatives.

Now let us turn our attention to the independence condition, and see how it relates with two previous conditions.

\(^{58}\) Arrow, “A Difficulty in the Concept of Social Welfare,” 336-337.

IIA: Let $R_1$, $R_2$, and $R'_1$, $R'_2$ be two sets of individual orderings. If, for both individuals $i$ and for all $x$ and $y$ in a given set of alternatives $S$, $xR_i y$ if and only if $xR'_i y$, then the social choice made from $S$ is the same whether the individual orderings are $R_1$, $R_2$, or $R'_1$, $R'_2$.\(^{60}\)

IIA sets a limit to the number of alternatives that individual members may choose from. It tells us that we cannot add to nor subtract from the initial set of alternatives since the very idea of a social welfare function would be theoretically inert.\(^{61}\) Arrow argues that

[t]he reasonableness of this condition can be seen by consideration of the possible results in a method of choice which does not satisfy Condition 3 (i.e., IIA), the rank-order method of voting frequently used in clubs.\(^{62}\)

Following Arrow’s cue, let us imagine a scenario that IIA does not hold.\(^{63}\) Let us assume that we can subtract from an initial set of four alternatives, $w, x, y, z$ that a three-member society may choose from.

Suppose that $a$ and $b$ have the same ranking profile where $x$ is more preferred than $y$, which, in turn, is more preferred than $z$, and $z$ is more preferred than $w$. Suppose further that $c$’s ranking profile has this order of preference ranking instead: $z, w, x$, and $y$. Given UD and PE, the social welfare function would yield $x$ as the clear social choice from the three profiles. And this is expected given that two out of three members have ranked it as first.

However, if we can subtract $y$ from the set of alternatives, the social welfare function generates a tie between $x$ and $z$.\(^{64}\) Thus, if IIA does not hold, the social welfare function cannot do its job of generating well-behaved social choices from individual preferences. Furthermore, without IIA, the result of any social deliberation would be a product of happenstance. As Arrow puts it, “the result... [is] dependent on the obviously accidental circumstance.\(^{65}\)
UD, IIA, and PE form a consistent set of conditions necessary for a social welfare function, a function, which makes social choice possible. A social welfare function necessarily has a fixed set of alternatives given IIA. It admits actual and possible individual preference ranking profiles given UD. And it generates social choice from these ranking profiles given PE.

Cashed out this way, UD, PE, and IIA are precisely the same conditions that make any system of voting possible. Voting requires that there are alternatives to choose from, individuals who signify their preferred alternative through a vote, and that a winning alternative is determined by a consensus. As described thus far, the necessary conditions for a social welfare function work the same way. Thus, as Arrow puts it, “in a generalized sense all methods of social choice are of the type of voting.”

Let us now discuss the non-dictatorship condition and see its relationship with the other three conditions.

**ND:** The social welfare function is not to be imposed or to be dictatorial.

ND is grounded on the idea that each individual is free to choose from a given set of alternatives, and that this free choice is not imposed. As it is stated, however, the condition that social choice should not be imposed is too strong a requirement since each individual preference is an imposition of the sovereignty of each individual. ND does not discount this facet of human choices. What it does is to prohibit one individual to impose what society as a whole will choose. That is, ND implies that no social welfare function must generate social choices from a single individual preference ranking.

All the other three conditions will fail to hold if ND does not hold. Suppose that the social welfare function yields \( x \) as the preferred alternative over some other alternative, \( y \). This means that, given UD, PE, and IIA, the aggregate of all individual ranking profiles places \( x \) as first and \( y \) as second. Now suppose that despite this, \( y \) was still instituted as the society’s choice. Then either the social welfare function failed to generate the right result or one individual preference became

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66 Cf. List, *op. cit.*, sec. 2.
67 Ibid., 338.
68 This formulation takes Arrow’s conditions 4 and 5 as one and the same. Cf. ibid., 337-338.
69 Cf. ibid., 338.
dictatorial. Now since, *ex hypothesi*, UD, PE, IIA hold, it follows that the social welfare function generated the right result. Thus, there is exactly one individual preference that became dictatorial.

Having discussed the four necessary conditions for social choice, we are now in a position to go through a simple version of Arrow’s proof. Recall that Arrow’s theorem tells us that social choice is impossible; i.e. no social welfare function satisfies all four conditions. Accordingly, this implies that: *if UD, PE, IIA are satisfied, then ND is not satisfied.*

In order to prove this, let us suppose, for *reductio*, that there is a social welfare function that satisfies UD, PE, IIA, and ND. Our aim is to show that this assumption leads to a contradiction.

Suppose that individuals in a society can choose between two alternatives $x$ and $y$. Then given UD, PE, IIA, and ND, a social welfare function generates a non-dictatorial social choice by aggregating all the admissible individual ranking profiles. Suppose that the social choice yields $x$. Then, given PE, a subset of all ranking profiles will be decisive. This means that some decisive group ranks $x$ as first and $y$ as second. This group might be a super majority, a simple majority, or a decisive minority, or whatnot. By iterating this process, we arrive at another decisive subgroup of the decisive group, and a sub-subgroup of the decisive subgroup, and so on, until we arrive at the decisive sub-sub-subgroup which contains only one individual member. Let $i$ be that individual. If $i$ is the sole member of that decisive group, then $i$ is a dictator. This contradicts, then, the assumption that a social welfare function is non-dictatorial. Thus, a non-dictatorial social choice is impossible.

Alternatively, Sen states the proof rather eloquently,

There is no non-dictatorial Arrovian social welfare function that would be satisfied simultaneously with the other three requisites. Given that the group of all persons is decisive. By virtue that in any decisive group containing more than one person there is a subgroup that is decisive without a support of the rest, it would follow that there is a possibility of reducing the decisive group as long as it contains more than one person. Since the set of individuals is finite, then we must in this way arrive at

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70 See fn. 55.
one person being decisive over all pairs, i.e. being a dictator. And that contradicts the non-dictatorship requisite.\textsuperscript{71}

**Summary and Replies to Objections**

In the second section of the paper, we have seen the *prima case* why choosing a distributive justice principle is impossible given Arrow’s theorem. The succeeding sections thereafter presented the case for the premises of the argument. In the third section, we provided reasons why choosing a distributive justice principle is a social choice. In the previous section, we presented Arrow’s proof of why social choice is impossible. In this final section, let us consider some objections to the argument in order to further clarify the main position developed in this paper.\textsuperscript{72}

**Objection.** Arrow’s impossibility theorem is a formal result. While the problem of distributive justice evidently has philosophical elements that could be analyzed by a formal science, like mathematics and logic, it is not by any means an exclusively formal problem.

**Reply.** The objection seems to be grounded on a rather faulty assumption that results in mathematics and logic cannot provide answers to philosophical problems that are not exclusively formal in character. Examples to the contrary abound in the literature: the mathematical analysis of continuous functions as a solution to Zeno’s paradoxes;\textsuperscript{73} treating beliefs as subjective probabilities as an answer to some epistemic problems;\textsuperscript{74} and, as we have argued here, Arrow’s results as applied to the problem of distributive justice.

**Objection.** It is quite mistaken to dismiss the entire idea of distributive justice altogether because of a mathematical result.

**Reply.** The argument presented here does not dismiss the idea of distributive justice. It actually tries to make sense of it in terms of the resources of social choice theory. Taken this way, the argument cashes out as a meta-theory of distributive justice such that it theorizes about theorizing about distributive justice. If distributive justice theories


\textsuperscript{72} My thanks to this journal’s referees for pointing out some of these objections.


provide norms for distribution, our meta-theory provides an explanation of what it is to provide such norms. It is easy to appreciate reasonable applications of social choice mechanisms and distributive justice principles in actual social and political contexts.

*Objection.* The argument presented here does not discount the fact that social choice mechanisms and distributive justice principles apply to actual contexts. Just as economists use social choice theory to explain consumer-preference behavior, political theorists use distributive justice theory to justify social policies. As was said before, however, our argument is a meta-theory that employs social choice theory to distributive justice theory. In effect, what we are trying to explain is how people in a democratic setting arrive at a decision of how benefits and burdens must be shared.

*Objection.* The argument’s conclusion seems to advance a rather pessimistic and even a nihilistic view of social choice and distributive justice, which is not only preposterous but also dangerous.

*Reply.* The conclusion that the argument drew from Arrow’s theorem does seem pessimistic; it may even be preposterous and dangerous. Perhaps, this incredulity is warranted. Following Sen’s insights, the argument’s conclusion does come at high cost since, if right, we have to let go of a participatory form of decision-making where individuals in a society may have a say on issues that affect society by casting their votes or expressing their opinions. Furthermore, we also have to do away with a welfare system where all “heterogeneous interests of a diverse population” are considered.

Instead of incredulity, perhaps we could take a more positive outlook towards the argument, and treat it as a challenge to come up with a better explanation of how we could arrive at a collective choice of distributive justice principles, which is neither imposed nor dictatorial. This project was already started by Arrow himself, and later by Sen and others, and the work still continues today.

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75 Compare this with the distinction between ethics and metaethics. If ethics provides norms of right action, metaethics theorizes about what it is to provide such norms. Cf. Matthew Chrisman, *What is this Thing called Metaethics?* (London: Routledge, 2016), xv.

76 See fn. 51

77 Acknowledgments: Earlier versions of this paper were presented at the following venues: Prof. Zosimo Lee’s 2004 MA seminar on Social Philosophy at the University of the Philippines-Diliman, the 2006 Philosophical Association of the Philippines National Conference held in Antipolo, Rizal, and Prof. Napoleon M. Mabaquiao, Jr.’s 2008 PhD seminar on Socio-Political Philosophy at De La Salle
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University. My thanks go to the attendees of those events for their comments and suggestions. Special thanks go to Jove Jim Aguas, Paolo Bolanos, Mark Joseph Calano, the late Rolando M. Gripaldo, Michael Roland Hernandez, Zosimo Lee, Napoleon M. Mabaquiao, Jr., Emmanuel Olores, Alvin Sario, and Benito Teehankee for interesting discussions on this topic throughout the years. Finally, my thanks go to Hazel T. Biana and to the editors and two referees of this journal for their invaluable feedback that greatly improved the present version of this paper.


